excluded it only remains to apply the test for determining whether in the proposed equations (5) and (6), the auxiliary cubic is necessary. The general quartic

$$x^4+ax^3+bx^2+cx+d=0$$

is solvable by quadratics, without the intervention of a cubic, as shown by Thomas Simpson, in his Algebra, in three cases, viz., when

$$c=\tfrac{1}{2}af, \text{ or } a\sqrt{d} \text{ or } 2\sqrt{df},$$

where, for brevity, $f=b-\frac{1}{4}a^2$. Considering equation (6),

$$\begin{array}{ll} a=+3 & d=-4 \\ b=-21 & f=-23\tfrac{1}{4} \\ c=-64 & \end{array}$$

whence $\frac{1}{2}af=-34\frac{7}{8}$; $a\sqrt{d}=3\sqrt{-5}$; $2\sqrt{df}=5\sqrt{31}$.

As none of these quantities equal c, the conclusion is that y, and similarly x, cannot be determined by *strictly quadratic methods*.

Also solved by *J. E. SANDERS*, and *G. B. M. ZERR*.

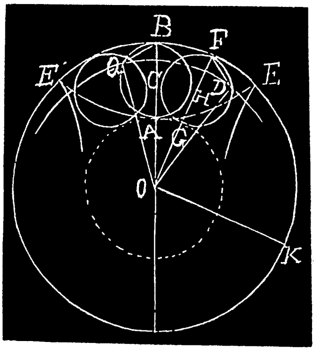
GEOMETRY.

191. Proposed by J. V. ADAMS, St. Louis, Mo.

Trisect any angle by means of the hypocycloid.

Solution by G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelyhia, Pa.; JOHN J. QUINN, Warren High School, Warren, Pa.; and M. E. GRABER, Heidelberg University, Tiffin, O.

Let EAE' be an arc of the hypocycloid from cusp to cusp; R=radius of fixed circle; r=radius of generating circle; AQB the central generating circle; C its center; Q any point on this circle. Join QB, QO. With QO as a radius and O as a center describe the arc QD meeting the hypocycloid in D. Let GDF be the generating circle when D is the generating point, GHF its diameter. Draw DF; construct angle BOK=to angle ACQ. Now arc GD=arc AQ, arc BQ=arc DF=arcFE.



$\therefore$ arc GD=arc BF=arc AQ. Arc BF measures angle BOF, arc AQ measures angle ACQ. But arc $BF=r/R$ arcBEK.

$\therefore$ Angle $BOF=r/R$ angle $BOK=r/R$ angle ACQ. If $R=3r$, angle $BOF=\frac{1}{3}$angle ACQ.

Therefore, by means of a suitable hypocycloid any angle may be divided in any given ratio. This property is applicable to the epicycloid also.

Also solved by the *PROPOSER*.